Indian Statistical Institute
Bangalore Centre
B.Math Second year
Semestral Examination (Supplementary)

Statistical Methods I 30.11.04

Answer as much as you can. The maximum you can score is 100
Time :- 3 1/2 hours

1. (a) Define sample mean and standard deviation of a random variable. For a set of 250 observations on a certain random variable X, the mean and standard deviation are 65.7 and 4.4 respectively. However, on scrutinizing the data it was found that two observations were incorrectly recorded as 71 and 83 in place of 91 and 80. Obtain the correct values of the mean and standard deviation.

   (b) Define mode of a random variable. Consider a continuous random variable X with having probability density $f(x)$, mode M and mean μ.

   (i) If $Y = aX + b$, then show that the mode of Y must be $aM_o + b$.

   (ii) Suppose for all values of $x > 0$, $f(x)$ satisfies the following inequality.

$$f(M - x) < f(M + x).$$

   Is $\mu >, <$ or $= M$? Justify.

   [6 + 4 + 8 = 18]

2. The aim of a study is to investigate the nature of dependence of the solubility of plutonium on temperature. From past experience, it was felt that log_{10}(solubility) was possibly linearly related with temperature and this relation was decided to be fitted. n different temperatures were chosen and log_{10}(solubility) of a plutonium compound in those temperatures were measured.

   (a) Derive the expressions for (i) least square estimate of the slope and (ii) the variance of this estimate.

   (b) Predict the solubility at a new temperature and construct a 90 /

   (c) Derive a test procedure for testing whether the slope is zero.

   [(6 + 8) + 6 + 8 = 28]

3. (a) Consider two independent random variable X_1 and X_2, X_i following Gamma $(\alpha, p_i), i = 1, 2$. Find the probality distributions of

   (i) $X_1 + X_2$, (ii)X_1/X_2 and (iii) $X_1/(X_1 + X_2)$.

   (b) Suppose X_i follows $\chi^2(k_i), i = 1, 2$ and that they are independent. Derive the probality density of $(X_1/k_1)/(X_2/k_2)$.

   [(6 + 6 + 8) + 5 = 25]

4. Suppose $X_1, X_2, \cdots X_n$ are i.i.d. random variables following $N(\mu, \sigma^2)$.

   (a) Assume $\mu = 0, \sigma = 1$. Derive the distributions of X_1^2 and $\sum_{i=1}^{n} X_i^2$.

   (b) Show that $\bar{X}$ and $s^2 = (1/(n-1)) \sum_{i=1}^{n} (Xi - \bar{X})^2$ are independent.

   (c) Derive the distribution of s^2.

[(5+3) + 9 + 5 = 22]

5. A student buys a lottery ticket for Rs.10. For every 1000 tickets sold, three winners are given bicycles costing Rs. 2000 each.

   (i) Find the probability of winning a bicycle.

   (ii) Determine the student's expected gain.

[2 + 5 = 7]

6. A manufacturer of furniture polish claims that its new product gives a more glossy finish than a leading brand. 18 houses are selected at random for comparing these two brands. Assuming that there is really no difference, find the probability that 13 or more houseowners will prefer the new product.

[8]